Life of Fred®
Beginning Algebra
Expanded Edition

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What Algebra Is All About

When I first started studying algebra in the ninth grade, no one in my family could explain to me what it was all about. My dad had gone through the eighth grade in South Dakota, and my mom never mentioned that she had ever studied algebra before she took a job at Planter's Peanuts in San Francisco.

My school counselor enrolled me in beginning algebra, and I showed up to class on the first day not knowing what to expect. On that day, I couldn't have told you a thing about algebra except that it was some kind of math.

In the first month or so, I found *I liked algebra better than* . . .

✓ physical education, because there were never any fist-fights in the algebra class.

✓ English, because the teacher couldn't mark me down because he or she didn't like the way I expressed myself or didn't like my handwriting or didn't like my face. In algebra, all I had to do was get the right answer and the teacher had to give me an A.

✓ German, because there were a million vocabulary words to learn. I was okay with der Finger which means *finger*. But besetzen, which means to occupy (a seat or a post) and besichtigen, which means to look around, and besiegen, which means to defeat, and the zillion other words we had to memorize were just too much. In algebra, I had to learn how to *do stuff* rather than just memorize a bunch of words. (I got C's in German.)

✓ biology, because it was too much like German: memorize a bunch of words like mitosis and meiosis. I did enjoy the movies though. It was fun to see the little cells splitting apart—whether it was mitosis or meiosis, I can't remember.

So what's algebra about? Albert Einstein said, "Algebra is a merry science. We go hunting for a little animal whose name we don't know, so we call it x. When we bag our game, we pounce on it and give it its right name."

What I think Einstein was talking about was solving something like $3x - 7 = 11$ and getting an answer of $x = 6$.

But algebra is much more than just solving equations. One way to think of it is to consider all the stuff you learned in six or eight years of studying arithmetic: adding, multiplying, fractions, decimals, etc. Take all of that and stir in one new concept—the idea of an "unknown," which we like to call "x." It's all of arithmetic *taken one step higher*.

Many, many jobs require the use of algebra. Its use is so widespread that virtually every university requires that you have learned algebra before you get there. Even English majors, like my daughter Margaret, had to learn algebra before going to a university.

I also liked algebra because there were no term papers to write. After I finished my algebra problems I was free to go outside and play. Margaret had to stay inside and type all night. A lot of English majors seem to have short fingers (die Finger?) because they type so much.

Life is unlivable if it is confined to algebra.

Life is incomplete without it.

Common Questions that Students Have

MAY I USE MY CALCULATOR?

Yes. It is the addition and multiplication tables that you need to know by heart. Once you have them down cold, and you know that the area of a triangle is one-half times base times height, there is little else that you should have to memorize.

When I taught arithmetic, the tests I gave were always taken without the use of a calculator, but when I taught algebra/geometry/trigonometry/calculus/math for business majors/statistics, the tests were always open-book, open-notes, and use-a-calculator-if-you-want-to.

There are a lot of times in life when you may need to know your addition and multiplication facts and won't have access to a calculator, but when you are doing algebra or calculus problems you will almost always have a calculator and reference books handy.

WHAT KIND OF CALCULATOR WOULD BE GOOD?

A basic calculator has these five keys: +, – , ×, ÷, $\sqrt{\ }$. Years ago I saw one of those advertised in a magazine for over $100. Recently, at one of those stores that sell everything for about a dollar I paid $1.07 including the sales tax.

Most top-rated universities want their applicants to have four years of high school math. (Beginning algebra is the first of those four years.)

The next three years will be advanced algebra, geometry, and trig. For those courses you will need a "scientific calculator." It will have sin, cos, tan, !, log, and ln keys. The most fun key is the "!" key. If you press 8 and then hit the ! key, it will tell you what 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 is equal to. Recently, I saw one of those calculators on sale for less than $12. That's the last calculator you'll need to learn all the stuff through calculus.*

You might as well get your scientific calculator now.

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* Some schools require their calculus students to buy a fancy graphing calculator that costs between $80 and $120. I don't own one and I've never needed one. I spent the money I saved on pizza.

WHAT BACKGROUND DO I NEED TO START ALGEBRA?

If you are feeling unsure, you might try your hand at this quiz. The answers are given on the next page.

The questions are all taken from books that precede *Life of Fred: Beginning Algebra Expanded Edition.*

Am I Ready for Algebra?

Use just pencil and paper.

1. $4\frac{2}{5} \div 3\frac{1}{3}$ and simplify your answer (from *LOF: Fractions*, p. 151)
2. If a 12-inch (diameter) pepperoni pizza costs \$9.48, how much would one square inch cost? Round your answer to the nearest cent. Use 3 for π. (from *LOF: Decimals & Percents*, p. 103)
3. A motorboat normally rents for \$71. If you don't sing a sea chanty in the store, you get a 30% discount. How much will the rental price be after the discount? (*LOF: D&P*, p. 112)
4. Is {(2, 3), (1, 4), (3, 3)} a function? (*LOF: D&P*, p. 167)
5. Of six pounds of sandwiches, Joe eats 98%, drops 1% overboard, and uses 0.5% for bait. How many *ounces* are left for the dog to eat? (*LOF: Pre-Algebra 1 with Biology*, p. 154)
6. If Kim can do 5 bank transactions in 8 minutes, how long would it take Kim to do 18 bank transactions? (*Life of Fred: Pre-Algebra 2 with Economics*, p. 26)
7. Is it possible for a function whose domain is {A, B, C} and whose codomain is {Y, Z} to be 1-1? (*Life of Fred: Pre-Algebra 2 with Economics*, p. 52)

The answers: (1) $1\frac{8}{25}$ (2) \$0.0877 which rounds to 9¢ (3) \$49.70 (4) Yes (5) 0.48 ounces (6) 28.8 minutes or $28\frac{4}{5}$ (7) No because functions that are 1-1 must have at least as many elements in the codomain as in the domain.

If you didn't get at least 70% (5 out of 7) right, the intelligent thing to do might be to start with one of the earlier books in the series.

WHERE ARE **THE BRIDGES**?

At the end of each chapter are three Cities. They are not tests. There is a lot of math to learn in this first year of high school math. The Cities offer a much-needed chance to practice your algebra. Do not skip any of them. By the time you do the third City in each chapter, you will be doing the problems much more easily.

One Important Rule

When you come to the questions in the *Your Turn to Play* or in the Cities, you must take out a piece of paper and write out the answers to the questions BEFORE you look at the answers.

At first, the questions will seem easy, and you will be tempted to ignore this rule. But by chapter four (word problems), students who have habitually just looked at the questions and read the answers will often find that they are "lost."

You will learn a lot more by working the problems.

Just before the Index is the **A.R.T.** section, which very briefly summarizes much of beginning algebra. If you have to review for a final exam or you want to quickly look up some topic eleven years after you've read this book, the **A.R.T.** section is the place to go.

A Note to Parents

Your children are now on "automatic pilot." Each day they do one (or more) lessons. The reading in *Life of Fred: Beginning Algebra Expanded Edition* is fun. And because it is fun, they will learn mathematics much more easily.

Five-year-old Fred *first* encounters the need for mathematics in his everyday life, and *then* we do the math. This is true for all of the books in the series. The math is *relevant*. This is different than most math books.

I believe that mathematics should not be taught in a vacuum. It should not be compartmentalized. We are teaching children first, not just math. Other subjects are integrated into the text. I have not taken the oath: "Algebra, the whole algebra, and nothing but the algebra."

In this book we include some **English**. Do you know the complete "i before e" rule with its four classes of exceptions? It's in this book. The army chaplain is at a private library and he pulls a leather-bound book of poetry off the shelf and begins to read a poem. He thinks to himself, "A good example of enjambment." This word is then defined in a footnote.

Health. Fred and Jack LaRoad decided to head out for an afternoon jog. The other eleven decided to watch TV for five hours. On another occasion, when Fred and Jack were on a six-hour army leave in a town they had never been in before, they headed to a carrot juice bar.

Reading. But before that, they went to the public library. "He loved books and had heard of this library from the chaplain on the army base. It has more than 22,000 books, magazines, and audio tapes. Fred's eyes and fingers were itching to examine them all."

Vocabulary. In telling the story of Fred's life, I use a full adult vocabulary, for example, the words *eponymous, hebdomadal,* and *faux pas*. However, the vocabulary is kept simple when I'm explaining the math.

Students are expected to do ALL of the problems. It is really better for them if you don't help them with any of the problems. It is so important that they learn how to learn by reading. If it takes them two days to figure out a particular problem, that is perfectly fine.

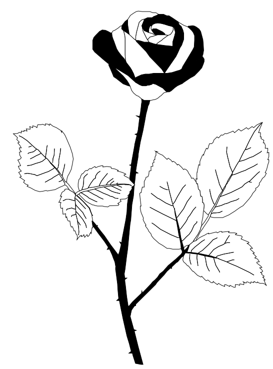
There is an old story of someone who saw a butterfly trying to break out of its chrysalis. He felt sorry for the effort that the butterfly was making and tried to "help" it by breaking open the chrysalis. The butterfly could never fly. It needed to struggle and exercise to develop its wings. (In *Life of Fred: Butterflies* we learned that butterflies do not use cocoons.)

Contents

Chapter One

Lesson One—Finite/Infinite, Exponents, and Counting

He stood in the middle of the largest rose garden he'd ever seen. The warm sun and the smell of the roses made his head spin a little. Roses of every kind surrounded him. On his left was a patch of red roses: *Chrysler Imperial* (a dark crimson); *Grand Masterpiece* (bright red); *Mikado* (cherry red). On his right were yellow roses: *Gold Medal* (golden yellow); *Lemon Spice* (soft yellow). Yellow roses were his favorite.



Up ahead on the path were white roses, lavender roses, orange roses and even a blue rose.

Fred ran down the path. In the sheer joy of being alive, he ran as any healthy five-year-old might. He ran and ran and ran.

At the edge of a large green lawn, he lay down in the shade of some tall roses. He rolled his coat up in a ball to make a pillow.

Listening to the robins singing, he figured it was time for a little snooze. He tried to shut his eyes.

They wouldn't shut.

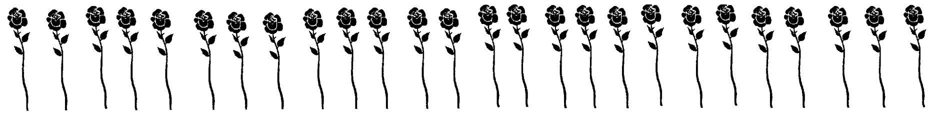
Hey! Anybody can shut their eyes. But Fred couldn't. What was going on? He saw the roses, the birds, the lawn, but couldn't close his eyes and make them disappear. And if he couldn't shut his eyes, he couldn't fall asleep.

You see, Fred was dreaming. He had read somewhere that the only thing you can't do in a dream is shut your eyes and fall asleep. So Fred *knew* that he was dreaming and that gave him a lot of power.

He got to his feet and waved his hand at the sky. It turned purple with orange polka dots. He giggled. He flapped his arms and began to fly. He settled on the lawn again and made a pepperoni pizza appear.

In short, he did all the things that five-year-olds might do when they find themselves King or Queen of the Universe.

Soon he was bored. He had done all the silly stuff and was looking around for something constructive to do. So he lined up all the roses in one long row.

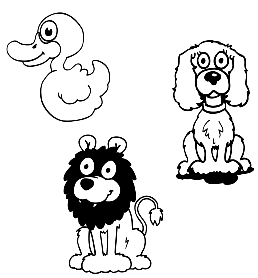


They stretched out in a line in both directions going on forever. Since this was a dream, he could have an unlimited (**infinite**) number of roses to play with.

When Fred was three years old, he had spent some time studying physics and astronomy. He had learned that nothing in the physical universe was infinite. Everything was **finite** (limited). Every object could travel only at a finite speed. Even the number of atoms was finite. One book estimated that there are only 10^{79} atoms in the observable universe. 10^{79} means 10 times 10 times 10 . . . a total of 79 times, which is 10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000, 000,000,000,000,000,000,000,000,000,000,000. That is a lot of atoms! (The "79" is an exponent—something we'll deal with later.)

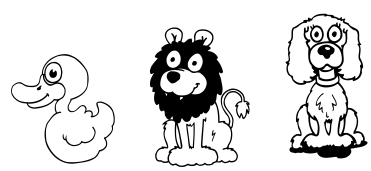
Now that he had all the roses magically lined up in a row, he decided to count them. Math was one of Fred's favorite activities.

Now, normally when you've got a bunch of stuff in a pile to count,



(← *These are Fred's dolls that he used to play with when he was a baby.*)

you line them up ....



and start on the left and count them 1 2 3

But Fred couldn't do that with the roses he wanted to count. There were too many of them. He couldn't start on the left as he did with his dolls. Dolls are easy. An infinite line of roses is hard.

Now it's your turn to play with some of the things covered thus far. Take out a piece of scratch paper and write out the answers for each of the following. **This is important.** They've done the studies and have found that you learn and retain a lot more if you are actively involved in the learning process rather than just reading passively.

Your Turn to Play

1. Is there a finite or infinite number of grains of sand on all the beaches in the world?

2. 10^{79} means 10 times 10 times 10 . . . seventy-nine times. What does 3^4 equal?

3. Which is larger: 2^5 or 5^2?

4. In Fred's dream the set (collection) of roses was infinite. The set of even natural numbers, {2, 4, 6, 8, 10, 12, 14, . . . }, is infinite. The set of all possible melodies is infinite.

    You don't find infinite sets at the grocery store.

    You don't find infinite sets in your laundry basket.

    Where *is* a good place to find infinite sets?

5. What does 1^{8369} equal?

6. What would you multiply 10^2 by in order to get 10^5?

Intermission

Some people like to argue that infinite sets don't really exist. "After all," they say, "they're just a figment of your imagination. It's all in your head."

By that same argument I could prove that pain doesn't exist. When you cut your finger, the pain is experienced in your brain.

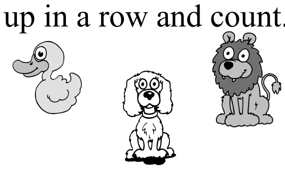
And the pleasure of a bite of warm pizza doesn't exist.

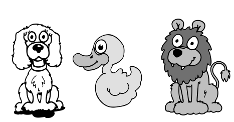
And the number three doesn't exist.

And truth doesn't exist.

Just because it is happening inside your skull doesn't mean that it doesn't exist.

7. When you want to count something, one of the easiest ways is to line them up in a row and count.





1 2 3

A hard question: Why doesn't it make a difference which order you line them up? Why do you always get the same answer?

. . . . COMPLETE SOLUTIONS . . . .

1. Nothing in the physical universe is infinite. There is a finite number of grains of sand.
2. $3 \times 3 \times 3 \times 3$ which is 81.
3. 2^5 is $2 \times 2 \times 2 \times 2 \times 2$ which is 32. 5^2 is 5×5 which is 25. So 2^5 is larger.
4. The set of roses in a dream, the set of even natural numbers, and the set of all possible melodies are all things that we can conceive. They are not things we can touch. To find infinite sets, one of the best places to look is your mind.
5. If you keep multiplying 1 times itself, you will always get an answer equal to 1.
6. $10^2 \times ? = 10^5$ is a restatement of the question.
   $100 \times ? = 100{,}000$.
   $100 \times 1000 = 100{,}000$
   $10^2 \times 10^3 = 10^5$
7. Wow. That is something that most people never think about. They would say that it's *obvious* that the way you line up the items won't affect how many there are.

   Could it be that it's obvious to them because that's what they've always experienced? But suppose the world were created a little differently. Suppose that the order in which you lined up the objects affected how many there were? Then everyone would go around saying that it's obvious that the way you line up objects affects how many there are. One of the enduring mysteries of mathematics is how well the stuff that goes on in our heads reflects what goes on out there in the "real world." *That didn't have to happen.*

One of the fun things I sometimes do in a calculus class (when we're studying infinite series) is to write on the board: $1 - 1 + 1 - 1 + 1 - 1 \ldots$ and ask the students what the sum is.

If I add together the pairs I get $(1 - 1) + (1 - 1) + (1 - 1) + \ldots$ which is $0 + 0 + 0 + \ldots$ which equals zero.

If I combine the second and third numbers together, the fourth and fifth numbers together, etc., I get $1 \underline{- 1 + 1} \underline{- 1 + 1} - \ldots = 1 + 0 + 0 + 0 + \ldots$ which equals one.

Index

You have mastered all of beginning algebra. Next comes:

- *advanced algebra*
- *geometry*
- *trigonometry.*

After two years of college calculus, you will be a junior and ready to declare that you are a math major.

Being a math major! Yes. That's a lot more fun than being:

❂ an English major—and writing long term papers

❂ a chemistry major—and getting acid burns in the lab

❂ a psychology major—and dealing with all the abnormals

❂ a history major—and learning all those dates.

It's your choice.